Rotation Invariant Indexing For Image Using Zernike Moments and R–Tree

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Abstract
The Zernike moment algorithm and R-Tree algorithm are known as state of the art in the recognition of images and in the multimedia database respectively. The methods of storing the images and retrieving the similar images based on a query image automatically are the problems in the image database. This paper proposes the method to combine the Zernike moments algorithm and the R–tree algorithm in the image database. The indices of images which are retrieved from the extraction process using Zernike moments algorithm are used as the multidimensional indices to recognize the images. The multidimensional indices of Zernike moments which are stored in the R–tree are compared to the magnitudes of Zernike moments of a query image for searching the similar images. The result shows that the combination of these algorithms can be used efficiently in the image database because the recognition accuracy rate using Zernike moments algorithm is 95.20%.

Keywords: image database, image recognition, image rotation, R–tree, Zernike moments

1. Introduction
A problem to recognize an object which has different orientation arises in pattern analysis. There are many techniques used to recognize the image, such as the gray level histogram and the gray level co-occurences matrices features which are used to identify the human skin irritation [1], and the two dimensional Gabor filter to obtain the palprint features [2]. Since Zernike moments have a rotation invariant property [3], the algorithm of Zernike moments could be used to analyze an image such as medical image which has different orientation. This idea was re-introduced in [4] by extracting the magnitudes of Zernike moments from an image. Zernike moments are orthogonal complex moments [4]-[7]. Different order i.e. 3, 8, 20, and 39 of Zernike moments yields 2, 5, 11, and 20 magnitudes of Zernike moments respectively. These magnitudes of each order can be utilized as an index in a multidimensional tree. On the other hand, R-tree was known as state of the art in multimedia databases [8]. It is based on B-tree. R-tree was evolving to R+-tree [9], R*-tree [10], P-tree [11], TR*-tree [12], Hilbert R-tree [13], X tree [14] and virtual memory aggregation tree (VMAT) [15]-[17], and still continues to be researched, but the R-tree remains the simplest one.

In this paper we proposed an idea how to combine the magnitudes of Zernike moments into the R-tree. An input image is processed using the algorithm to yield its magnitudes. The
magnitudes of Zernike moments then are stored in an image database using the algorithm of R-tree. The algorithm of R-tree in the image database is used to store and to search the magnitudes of Zernike moments. To search the similar images in the image database, the magnitudes of Zernike moments of a query image are compared to the magnitudes of Zernike moments of the images which are stored in the R-tree [18].

2. Research Method

In this section the Zernike moments and the R-tree which are used to implement the image database will be explained.

2.1. The Zernike Moments

The Zernike moments are chosen to implement the image database because they have a rotation invariant property. The magnitudes of Zernike moments of a rotated image are invariant to the magnitudes of Zernike moments of an original image. The digital image of \( f(x, y) \) are mapped to the area of the unit circle which has the center of gravity \((x_c, y_c)\) in the polar coordinate system [4]. The center of gravity is calculated using the geometric moment function.

The Zernike polynomials that were introduced by F. Zernike in 1934 [3] are a set of orthogonal polynomials over the interior of the circular area. The definition of the Zernike polynomials is in the following form [4][6].

\[
V_{nm}(x, y) = V_{nm}(\rho, \theta) = R_{nm}(\rho)e^{jm\theta} \tag{1}
\]

where

- \( n \): Non-negative integer
- \( m \): Positive or negative integer which has the condition \( n - |m| \) is even and \( |m| \leq n \)
- \( (x, y) \): Position of the pixel in the Cartesian coordinate system
- \( j \): Imaginary unit
- \( \rho \): Length of vector from a position of the pixel \((x, y)\) which has the condition of \( \rho \leq 1 \)
- \( \theta \): Angle between vector \( \rho \) and the x axis in counterclockwise direction

\( R_{nm}(\rho) \) is the radial polynomial which has the following form

\[
R_{nm}(\rho) = \sum_{s=0}^{(n-|m|)/2} (-1)^s \frac{(n-|m|)!}{s!(\frac{n+|m|}{2})!\frac{(n-|m|)!}{s!(\frac{n-|m|}{2})!}} \rho^{n-2s} \tag{2}
\]

The Zernike moments for the image \( f(x, y) \) is defined as [4]

\[
A_{nm} = \frac{n+1}{\pi} \int_0^{2\pi} \int_0^1 f(\rho, \theta) R_{nm}(\rho)e^{-jm\theta} \rho \, d\rho \, d\theta \tag{3}
\]

The rotation invariant property of the Zernike moments can be expressed by [4]

\[
A_{nm}^{rot} = \frac{n+1}{\pi} \int_0^{2\pi} \int_0^1 f(\rho, \theta - \beta) R_{nm}(\rho)e^{-jm\theta} \rho \, d\rho \, d\theta \tag{4}
\]

If \( \theta_{rot} = \theta - \beta \) and \( \theta = \theta_{rot} + \beta \) then \( A_{nm}^{rot} = A_{nm}e^{-jm\beta} \)

The magnitudes of Zernike moments of the rotated image are invariant to the magnitudes of Zernike moments of the original image.

2.2. R-Tree Data Structure

Data structure that can handle multidimensional data index is needed in the process query of image data in a database system. The R-tree data structure will be used to handle multidimensional data index. The data pointer pointing to the image data in the form of a multidimensional index is located at the leaf node of R-tree. Leaf node R-tree has the same level as the structure of B-tree has a balanced high. Non Leaf node is pointing to a child that has the smallest rectangle or minimum bounding rectangle (MBR) [8]. Magnitudes of Zernike moments \( |A_{nm}| \) are used as an index of multidimensional data.
Table 1 shows an example of the magnitude of Zernike moments from a result of Zernike moments extraction process from nine images. Two parameters are used as the key index.

Table 1. The value of parameter magnitude Zernike

| Images No. | \(|A_3|, |A_6|\) |
|------------|----------------|
| 11         | (9.8, 4.9)    |
| 12         | (4.9, 12.8)   |
| 13         | (7.5, 14.4)   |
| 14         | (12.5, 10.3)  |
| 15         | (9.2, 12.3)   |
| 16         | (7.2, 10.3)   |
| 17         | (9.8, 12.8)   |
| 18         | (4.9, 7.5)    |
| 19         | (12.8, 9.2)   |

Figure 1. The illustration of an R-tree structure after insertion of nine indices

Figure 2. The relationship between coverage rectangles represented in Cartesian coordinate

Figure 1 and Figure 2 is an illustrative example of the preparation of an R-tree after nine key indices of two-dimensional (I₁, I₂, I₃, ... and I₉) is inserted in the R-tree and in the form of Cartesian coordinates. Number of maximum index in a node tree R is determined as much as three. The index is previously inserted in sequence one by one from the smallest index to the largest index number.

Value of error tolerance (εᵣ) is used to measure the similarity between two images. Error tolerance (εᵣ) value between the original image (S) index and the query image (Q) is defined as follows [19]:

\[
\varepsilon_R = \sqrt{\sum_{i=1}^{d} |Q_i - S_i|^2}
\]

where \(S_i\) is the index of the i-th dimension of the original image data stored, \(Q_i\) is the index of the i-th dimension of the query image, \(d\) is the number of dimensions in a single index.

The storage indices and the query index will be considered similar if \(\varepsilon_R (S, Q) \leq t\), \(t\) is the value of a predetermined error tolerance.

3. Results and Analysis

The effectiveness of image database system can be seen from the high average value of image recognition and the small number of output images obtained from the results of the comparison process Zernike parameters from input query image with the existing image data in the storage database system [18].
The image recognition is the criterion at the time of image data obtained from the search results similar with the input image query. The number of the image output is the number of images obtained from the search process the data in the image database system for one-time query. Recognizable images of query image A0 are shown in Figure 3. Image A0 is an image of magnetic resonance imaging (MRI) of head with the same section but has a different orientation of 60 and 90 degrees. The images which are not recognized are shown in Figure 4, where the shape of this image is a form of cross-sectional image of the MRI with a different head. Table 2 shows the error tolerance distance between the image of the recognized and unrecognized image in Figure 3 and Figure 4.

Table 2. The error tolerance distance between two images

<table>
<thead>
<tr>
<th>Image</th>
<th>$\varepsilon_R$</th>
<th>Image</th>
<th>$\varepsilon_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image A0 dan Image A90</td>
<td>0</td>
<td>Image C dan Image D</td>
<td>4.31</td>
</tr>
<tr>
<td>Image A0 dan Image A60</td>
<td>0.33</td>
<td>Image B dan Image A90</td>
<td>2.32</td>
</tr>
<tr>
<td>Image A60 dan Image A90</td>
<td>0.33</td>
<td>Image B dan Image A60</td>
<td>2.11</td>
</tr>
<tr>
<td>Image A0 dan Image B</td>
<td>2.32</td>
<td>Image C dan Image A90</td>
<td>1.65</td>
</tr>
<tr>
<td>Image A0 dan Image C</td>
<td>1.65</td>
<td>Image C dan Image A60</td>
<td>1.82</td>
</tr>
<tr>
<td>Image A0 dan Image D</td>
<td>3.41</td>
<td>Image D dan Image A90</td>
<td>3.41</td>
</tr>
<tr>
<td>Image B dan Image C</td>
<td>3.41</td>
<td>Image D dan Image A60</td>
<td>3.11</td>
</tr>
<tr>
<td>Image B dan Image D</td>
<td>2.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5(a) and Figure 5(c) show the recognition accuracy rates for rotated query images, with rotation angles of 30° and 60°, respectively. Figure 5(b) and Figure 5(d) show the retrieval number of images from the image database for a rotated query images, with rotation angles of 30° and 60°, respectively. The orders of Zernike moments which are used in the experiment are order 3, order 8, order 20 and order 39. The number of images which is use in the experiment is 1100 images. The error tolerance value which has the following condition: $0 \leq \varepsilon_R \leq 50$ is used in the experiment.

The output images will be retrieved by comparing the magnitudes of Zernike moments of a query image and the magnitudes of Zernike moments of the images in the image database. The error tolerance value between images the magnitudes of Zernike moments of a query image and the magnitudes of Zernike moments of the output images has to fulfill a condition subject to the following constraint: $\varepsilon_R (S,Q) \leq t$.

Figure 5(a) and Figure 5(c) give the information that the recognition accuracy rate for rotated query images is above 80% and the error tolerance value has the following condition: $2 \leq \varepsilon_R \leq 5$. It means that the magnitudes of Zernike moments of a rotated query image are invariant to the magnitudes of Zernike moments of the original image. Figure 5(a) and Figure 5(c) also give the information that the recognition accuracy rate for order three of Zernike moments for rotated query images is above 90% and the error tolerance value is equal to zero $\varepsilon_R = 0$. Figure 5(b) and Figure 5(d) give the information that the retrieval number of images from the image database for a rotated query images is above 20 images. It means that the Zernike moments of the order three cannot be used to retrieve the images from image database because the retrieval number of images has the value more than 20.
Figure 5. The graphics of the recognition accuracy rates and the retrieval number of images from the image database for rotated query images

The recognition accuracy rate of these experiments are compared to the recognition accuracy for the query image with noise free using generalized pseudo–Zernike moments algorithm [6]. The recognition accuracy rate for the experiments using Zernike moments algorithm for query images with rotation angle of 0° is 100%. The recognition accuracy of the second experiment for the query image with noise free using generalized pseudo–Zernike moments algorithm which was introduced by Xia et al [6] is 100%.

The image database could not be used efficiently if the error tolerance value is high. The higher the error tolerance value the bigger the number of images retrieved from image database. The recognition accuracy rate for all experiments using Zernike moments algorithm is 95.20%.

4. Conclusion

The Zernike moments algorithm and the R-tree algorithm could be used to implement the image database. The algorithm of Zernike moments is used to analyze the pattern of images such as medical images. The Zernike moments has a rotation invariant characteristic because the magnitudes of Zernike moments of the rotated image are invariant to the magnitudes of Zernike moments of the original image. The R-tree algorithm is used to store and to search the
magnitude of Zernike moments. The image database could not be used efficiently if the error
tolerance value is high because there are a lot of images retrieved from the image database.

References